

## Plenary panel

### **From Newton's first to second law: How can curriculum, pedagogy and assessment celebrate a more dynamic experience of calculus?**

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#### **Outline**

Calculus is a formidable toolbox for the study of change. Yet, at a time when digital technologies provide the capacity to create and celebrate dynamic experiences of calculus, institutional and other challenges may impede embracing this capacity in its curriculum, pedagogy and assessment. In many high stakes assessment systems, for example, coursework, formative testing and closed-book examination tasks seem to be stuck in the pre-digital age. Systemic inertia seems to manifest itself in other ways too. Calculus is needed in different shapes and forms in the different disciplines and professions; yet, it is typically introduced to those who study disciplines other than mathematics without due regard to the needs of the discipline. And, even though students often find calculus challenging and irrelevant—and, consequently, may disengage—it is still offered, unchanged, to them devoid of the *raison d'être* for its uses in their disciplines. The panelists will first, and briefly, share their experiences in the study and design of curriculum and assessment materials for calculus. Then, in the second and longer part of the panel session, through examples from those experiences, they will map out one possible way of fostering change: designing tasks—for classroom activity as well as assessment—that convey important meanings of calculus, are accessible, celebrate its dynamism, and are tailored to the needs of students in various disciplines who will soon enter diverse worlds of work.

#### **Provisional timeline**

Following a brief introduction by the moderator (3m), each panelist will address aforementioned current state of affairs ("*Inertia*") for about 10m each (30m). A first round of audience interventions, collated before the event, as well as on the spot possibly via a shared link, will follow (15m). The moderator will then briefly pull together the threads of the discussion so far (2m) and a second round of contributions from each panelist on how to foster change ("*Force and acceleration*") for about 15m each (45m). A further round of audience interventions (15m) will ensue and the session will close with a final comment from each panelist and the moderator (10m).

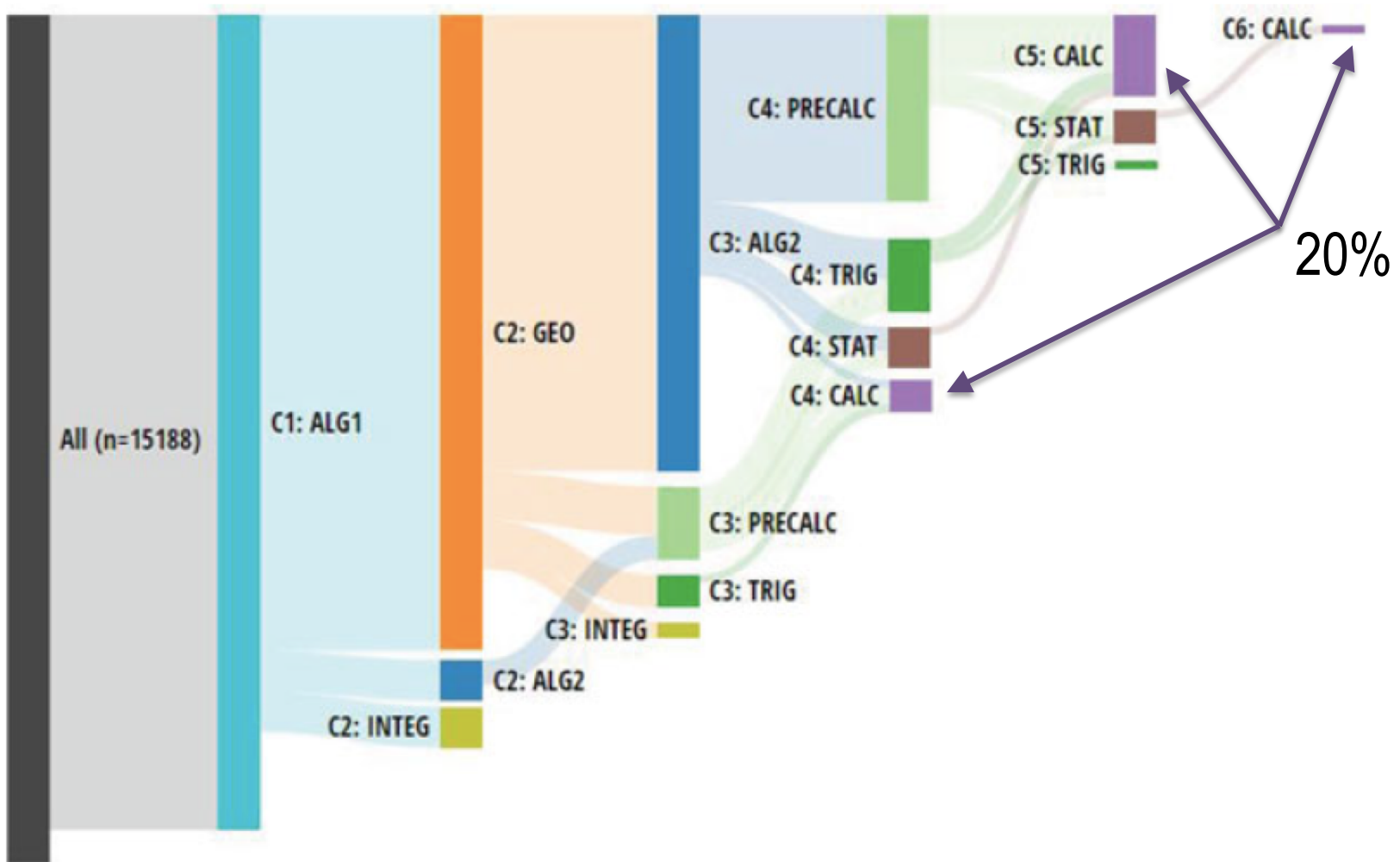
# CURRENT STATE OF AFFAIRS

## VILMA MESA

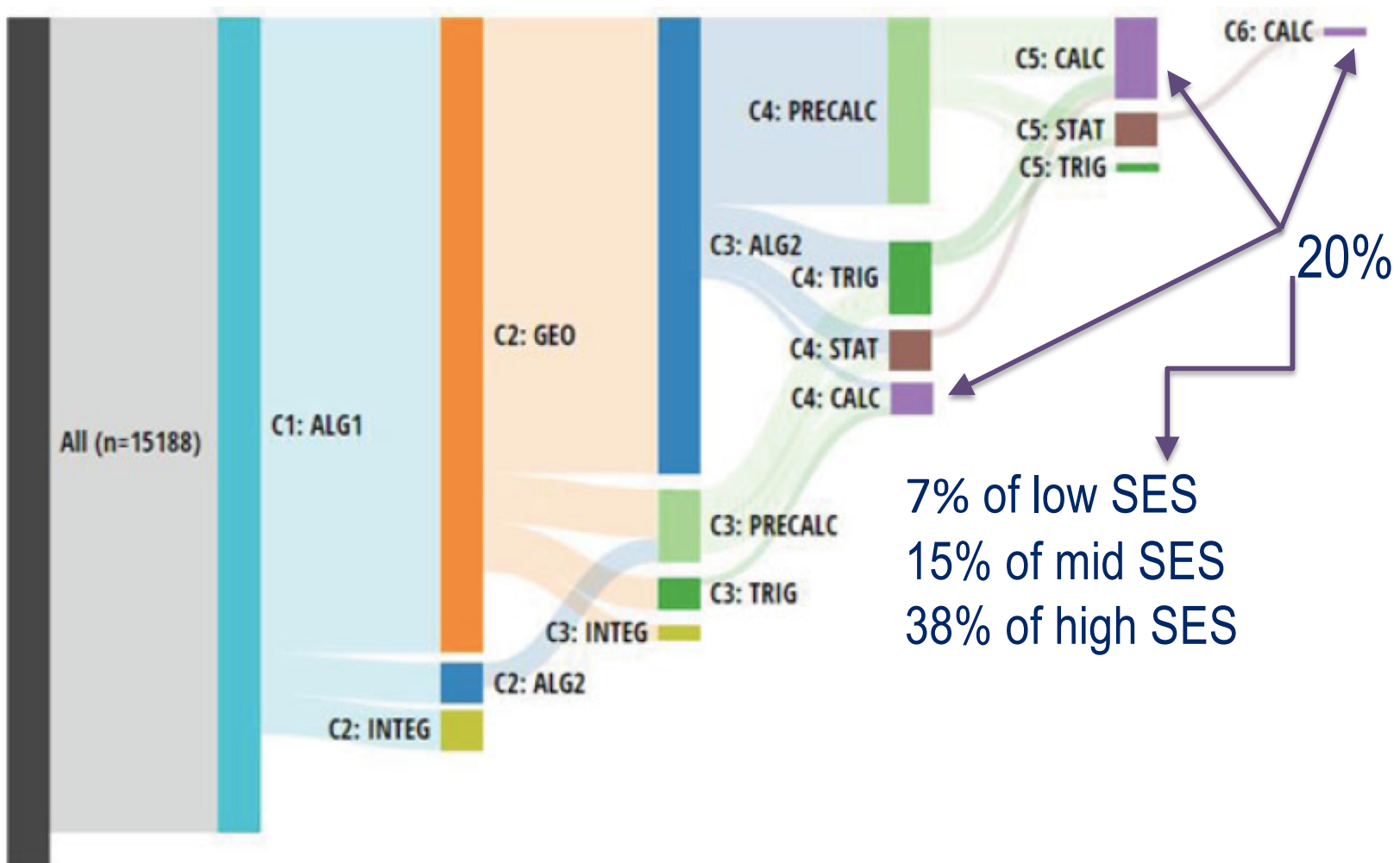


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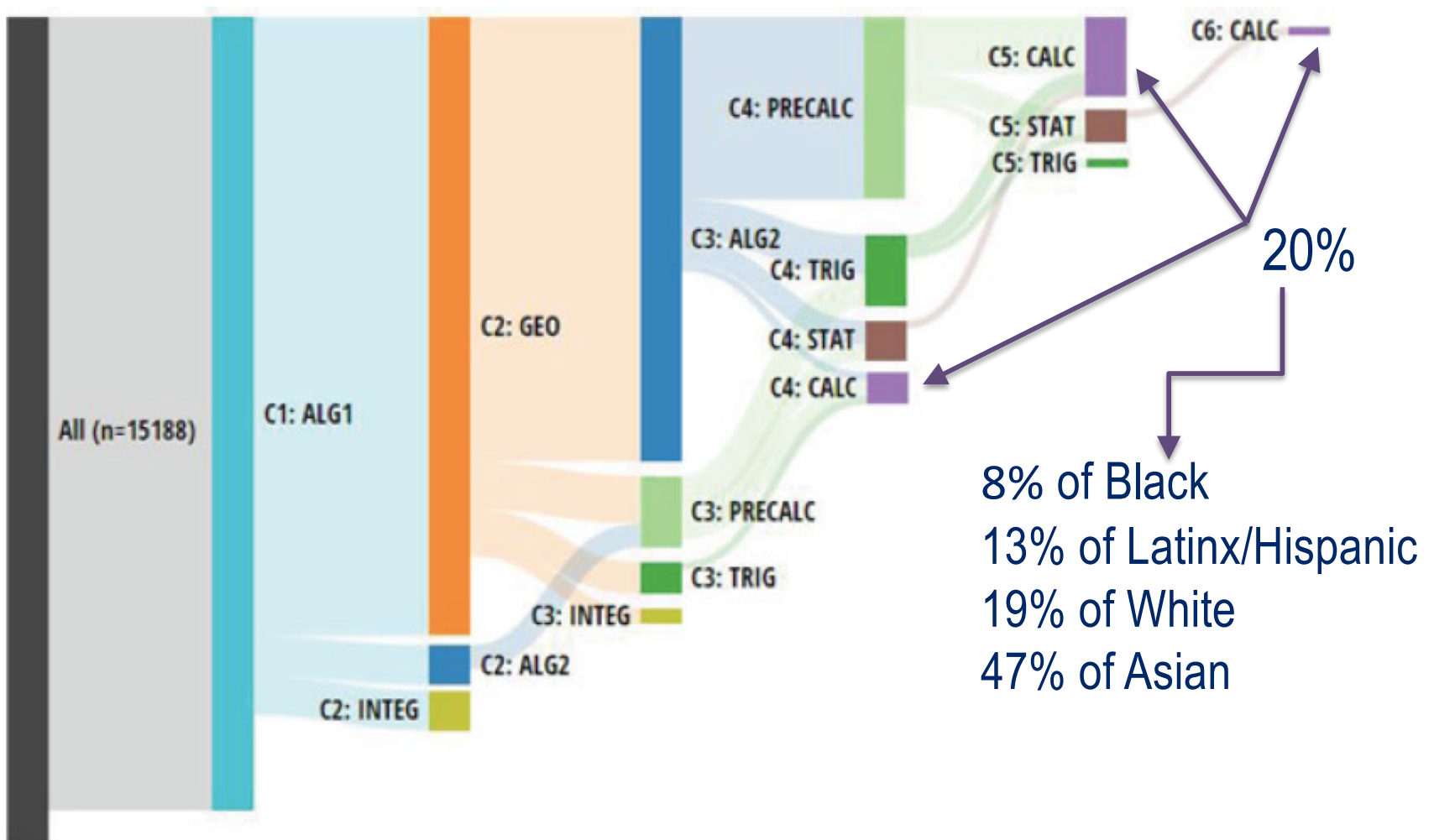
# CALCULUS CURRICULUM IN THE U.S.A.: WHO TAKES CALCULUS IN HIGH SCHOOL?



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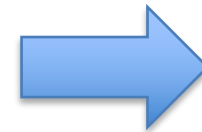


# CALCULUS CURRICULUM IN THE U.S.A.: WHO TAKES CALCULUS IN HIGH SCHOOL?



# CALCULUS CURRICULUM IN THE U.S.A.: WHICH MAJORS REQUIRE CALCULUS?

- Mathematics and Secondary Teaching
- Physics and Engineering
- Biology, Chemistry, other Natural Sciences  
(+ Medicine, Pharmacy, Kinesiology, and in some places Nursing)
- Computer and Information Science
- Psychology
- Statistics
- Economics, Business, and Accounting



Survival of  
math  
departments

# CALCULUS I CURRICULUM IN THE U.S.A.: WHAT IS THE CONTENT?

“Basic” differential and some integral calculus

- Limits and continuity (but no  $\delta$ - $\varepsilon$  proofs)
- Derivatives: concept, computation, representations, applications (less common L'Hôpital, Euler approx.)
- Integrals: indefinite, definite, applications, fundamental theorem of calculus
- Some courses (e.g., Advanced Placement):  
Differential equations and sequences and series

# CALCULUS TEACHING IN THE U.S.A.: WHAT DO WE KNOW?

## Promoted skills:

- Using of the “rule of four:” multiple representations in any situation
- Interpreting Variation/Covariation/Change
- Solving applied problems over demonstrating competency with algorithms
- Visualizing with technology
- Developing “employability skills:” communication and collaborative work

(Burn & Mesa, 2015; Hillel, 2001; Houston, 2001)



# CALCULUS TEACHING IN THE U.S.A.: WHAT DO WE KNOW?

Observed:

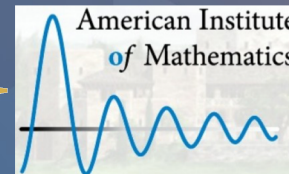
- Competency with complex symbolic manipulations
- Few applications in a lesson
- Non-existent technology use
- Limited opportunities for communication and collaborative work

VILMA MESA

# FOSTERING CHANGE



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# TEXTBOOK DESIGN

- Open source
- Open access
- Written in PreTeXt
- Interactive features
- Content organization

# TEXTBOOK AND USAGE TRACKING

- [Active Calculus](#)

By Matt Boelkins (<https://activecalculus.org/>)

- Tracking

<https://books.aimath.org/tracktest/>

# TEXTBOOK ORGANIZATION/SECTION

- Motivating questions
- Preview activities
- “Explanation” text with elaboration, definitions, theorems, demonstrations, commentary, invitations to explore with Geogebra applets
- Activities
- Reading questions (only for research purposes)
- Summary
- Exercises (WeBWork) and problems

# “EXPLANATIONS”

- “An excellent way to explore how the graph of  $f(x)$  generates the graph of  $f'(x)$  is through a java applet.” See, for instance, the applets at <http://gvsu.edu/s/5C>

or

<http://gvsu.edu/s/5D>

# WHAT WE HAVE LEARNED: SELF-REPORTED (+HEATMAP) USE

- Instructors...
  - Assign motivating questions and pre-activities as homework to be submitted prior to class
  - Depending on what students have done, may (not) do all activities
  - Most of the activities are done in class
- Students
  - Most say they do what their instructors ask them to do: pre-activities, classroom activities, WeBWork, homework

# WHAT WE HAVE LEARNED: RESOURCES AND WAYS OF USING TEXTBOOK

- Users reach to many resources
  - **Instructors:** old textbooks, past class notes (as students), activities they have used in the past, colleagues
  - **Students:** Wolfram Alpha, Google, tutors, Khan Academy, peers, instructor, YouTube
- Users view/use the textbook in many ways
  - Skim → activities → exercises → problems → read
  - Read explanations → take notes → examples → homework



# NEXT STEPS

1. Improving the representation of viewing data → Instructor dashboard?
2. Collecting data from users' mathematical work → complete feedback loop!
3. Analyzing conditions regarding various levels of classroom implementation

# UNDERGRADUATE TEACHING (AND LEARNING) IN MATHEMATICS WITH OPEN SOFTWARE AND TEXTBOOKS 3.0

THANK YOU!

Collaborators:

Rob Beezer	University of Puget Sound
David Farmer	American Institute of Mathematics
Tom Judson	Stephen F Austin State U
Susan Lynds	University of Colorado at Boulder
Yannis Liakos	University of Michigan-Ann Arbor
Kent Morrison	American Institute of Mathematics

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Partial support for this work was provided by the National Science Foundation's Improving Undergraduate STEM Education (IUSE) program under Award No. 1626455. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.

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