UTMOST 2

Undergraduate Teaching of Mathematics with Open Software and Textbooks

Improving Undergraduate STEM Education, Development and Implementation, Engaged Student Learning

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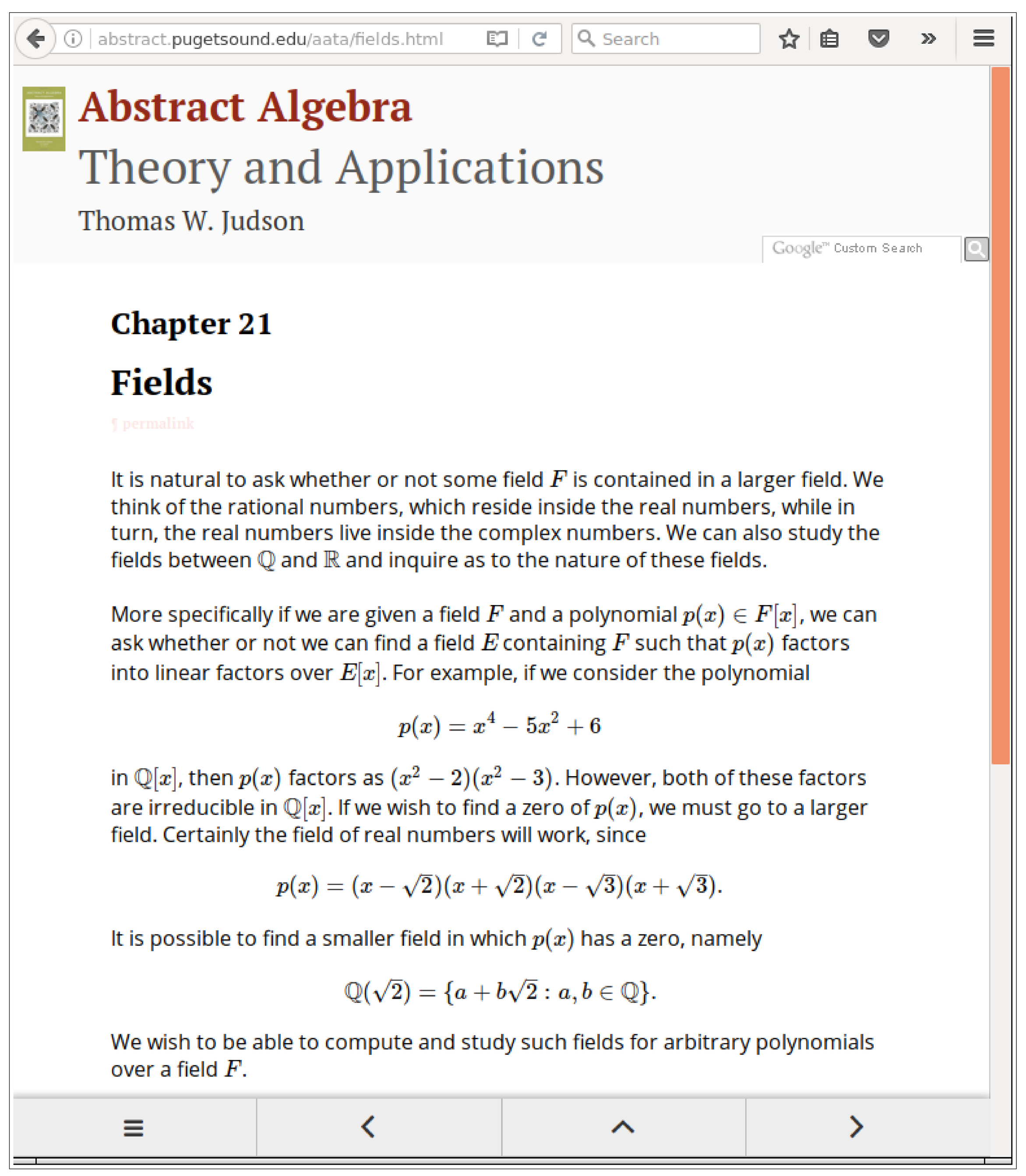
Electronic and Print Textbooks


abstract.pugetsound.edu/aata/fields.html

Abstract Algebra
Theory and Applications
Thomas W. Judson

Chapter 21

Fields

It is natural to ask whether or not some field F is contained in a larger field. We think of the rational numbers, which reside inside the real numbers, while in turn, the real numbers live inside the complex numbers. We can also study the fields between $\mathbb{Q}$ and $\mathbb{R}$ and inquire as to the nature of these fields.

More specifically if we are given a field F and a polynomial $p(x) \in F[x]$, we can ask whether or not we can find a field E containing F such that $p(x)$ factors into linear factors over $E[x]$. For example, if we consider the polynomial

$$p(x) = x^4 - 5x^2 + 6$$

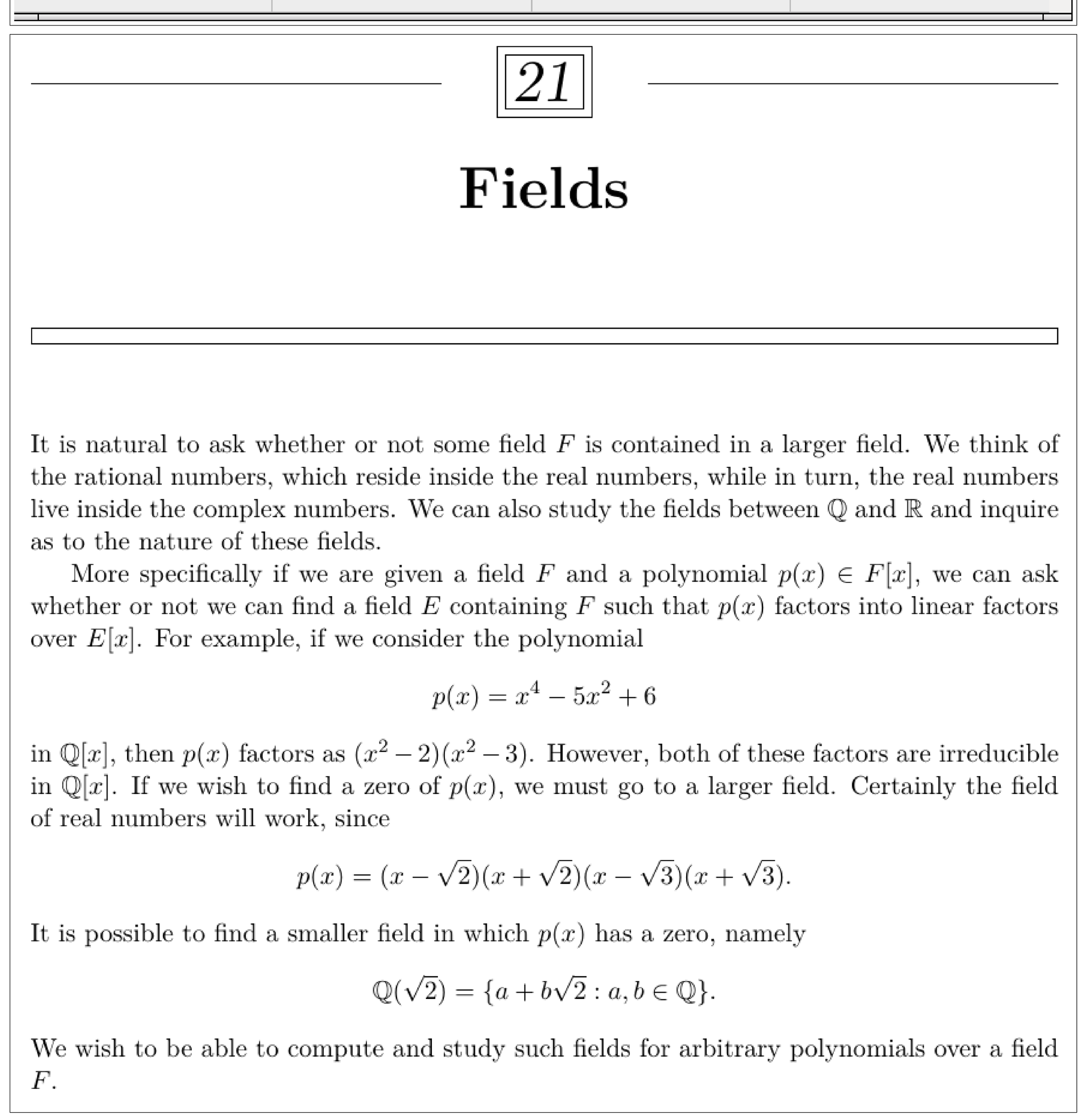
in $\mathbb{Q}[x]$, then $p(x)$ factors as $(x^2-2)(x^2-3)$. However, both of these factors are irreducible in $\mathbb{Q}[x]$. If we wish to find a zero of $p(x)$, we must go to a larger field. Certainly the field of real numbers will work, since

$$p(x) = (x-\sqrt{2})(x+\sqrt{2})(x-\sqrt{3})(x+\sqrt{3}).$$

It is possible to find a smaller field in which $p(x)$ has a zero, namely

$$\mathbb{Q}(\sqrt{2}) = \{a + b\sqrt{2} : a, b \in \mathbb{Q}\}.$$

We wish to be able to compute and study such fields for arbitrary polynomials over a field F.


21

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MathBook XML

- Separate structure and content from presentation
- Obtain multiple outputs from single source
- Markup with author-friendly XML, process with XSL stylesheets
- Output: Highly interactive web pages with sophisticated navigation
- Output: High quality print and PDF via LaTeX

Education Research Study

- Describe teacher and student use of electronic and print textbooks, inside and outside of the classroom
- Two open electronic textbooks, linear algebra and abstract algebra
- Sixteen courses at eleven institutions (variation by research intensity, size, location)
- Develop instruments and processes for data collection via:
  - a full term of teaching with periodic logs filled by teachers and students
  - an in-situ visit for observation and discussion of textbook use
  - beginning and end of term assessment of student growth (mathematical maturity and course content knowledge)
  - a match of data collected automatically with observations of use
- Document similarities and differences between uses of electronic and identical (PDF or print) versions
- Design recommendations based on uses of the textbooks

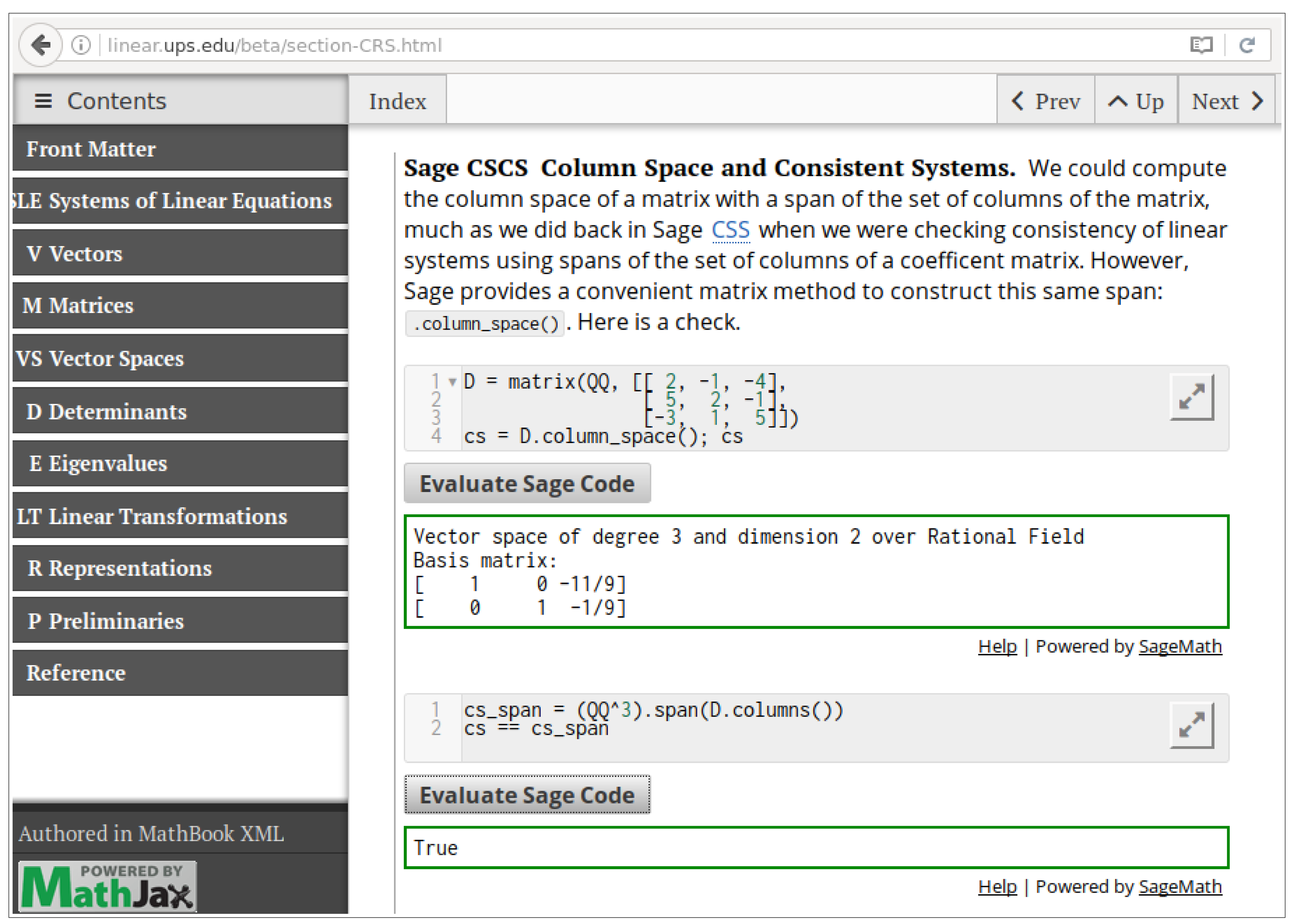
American Institute of Mathematics' Open Textbook Initiative

- 44 vetted and approved undergraduate mathematics textbooks
- Editorial Board
- Evaluation Criteria
- Author's Guide

Workshop

May 2017, Tacoma: "The integration of online materials and online textbooks"

Embedded Sage Cells


linear.ups.edu/beta/section-CRS.html

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Authored in MathBook XML

Sage CSCS Column Space and Consistent Systems. We could compute the column space of a matrix with a span of the set of columns of the matrix, much as we did back in Sage CSS when we were checking consistency of linear systems using spans of the set of columns of a coefficent matrix. However, Sage provides a convenient matrix method to construct this same span: `.column_space()`. Here is a check.

```
D = matrix(QQ, [[ 2, -1, -4],
                [ 5,  2, -1],
                [-3,  1,  5]])
cs = D.column_space(); cs
```

Evaluate Sage Code

```
Vector space of degree 3 and dimension 2 over Rational Field
Basis matrix:
[    1     0 -11/9]
[    0     1  -1/9]
```

Help | Powered by SageMath

```
cs_span = (QQ^3).span(D.columns())
cs == cs_span
```

Evaluate Sage Code

```
True
```

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Evaluation

External project evaluation provides formative and summative evaluation on processes, program events, research implementations, and participant feedback, using survey, interview, SMC usage, and observational data.

Advisory Board

- Jason Grout, Scientific Software Developer, Bloomberg LP
- Kiran Kedlaya, Professor, U of California, San Diego
- Gavin LaRose, Lecturer, Instructional Technology Manager, U of Michigan
- William Stein, Professor, U of Washington; CEO, Founder, SageMath, Inc.

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