

# UTMOST

## Undergraduate Teaching of Mathematics with Open Software and Textbooks

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$$\text{Free Open Source Math Software} + \text{Free Open Source Textbooks \& Curriculum} = \text{Wide integration of technology Driven by teachers themselves}$$

### Project Overview

“Our proposition is that freely-available open software, open textbooks, and other open curricular materials can allow teachers everywhere to transform the undergraduate mathematics curriculum by tightly and seamlessly integrating mathematics software with more traditional curricular materials.”

### Components

**Sage: Open Source Software for Mathematics**  
“Creating a viable free open source alternative to Magma, Maple, Mathematica and Matlab.”  
Includes over 100 open source packages  
Over 300,000 lines of new Python code

**Sage Servers: Web User Interface**  
Notebook: Javascript application in a web browser  
Server: runs locally, or on Linux or Mac servers

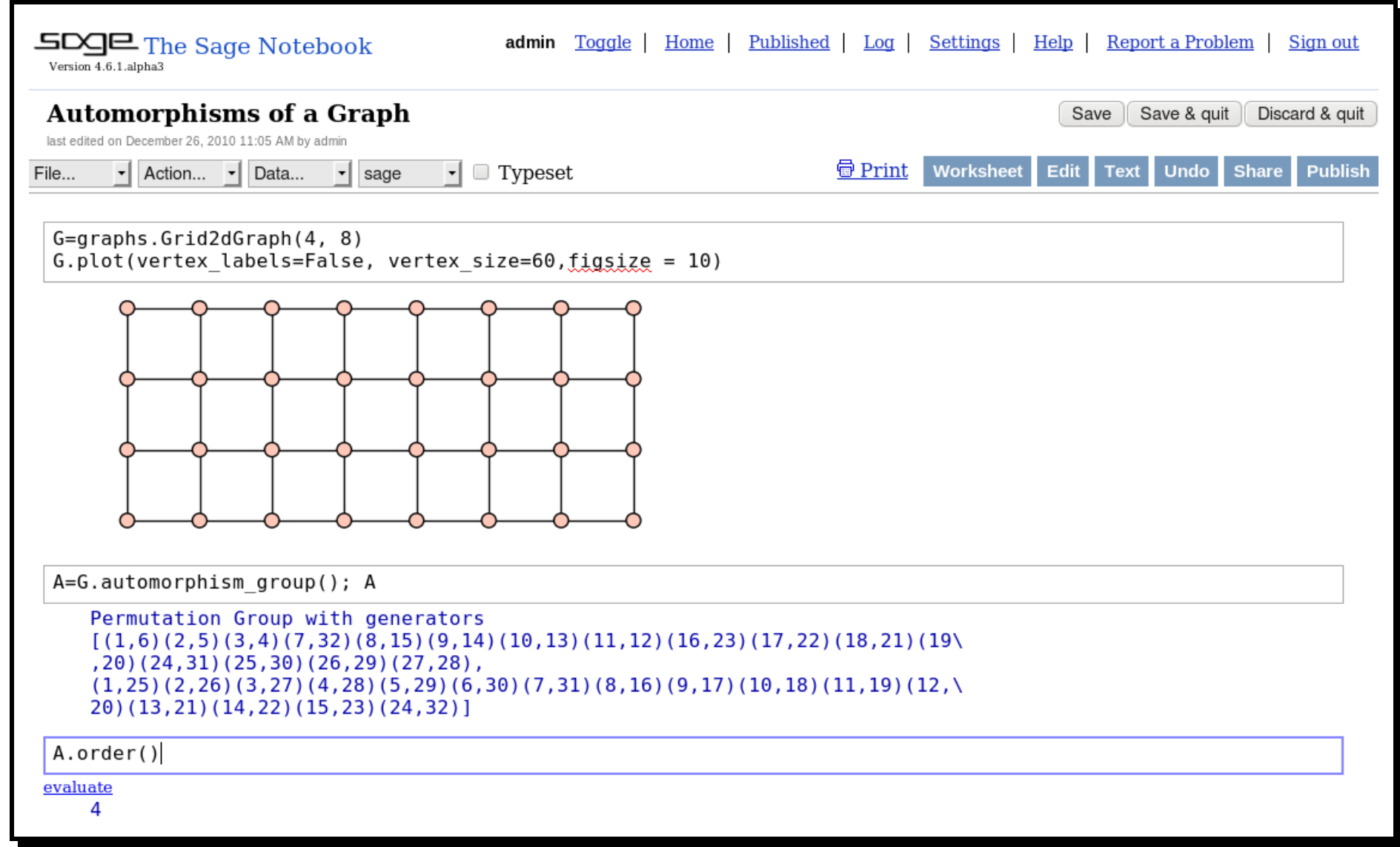
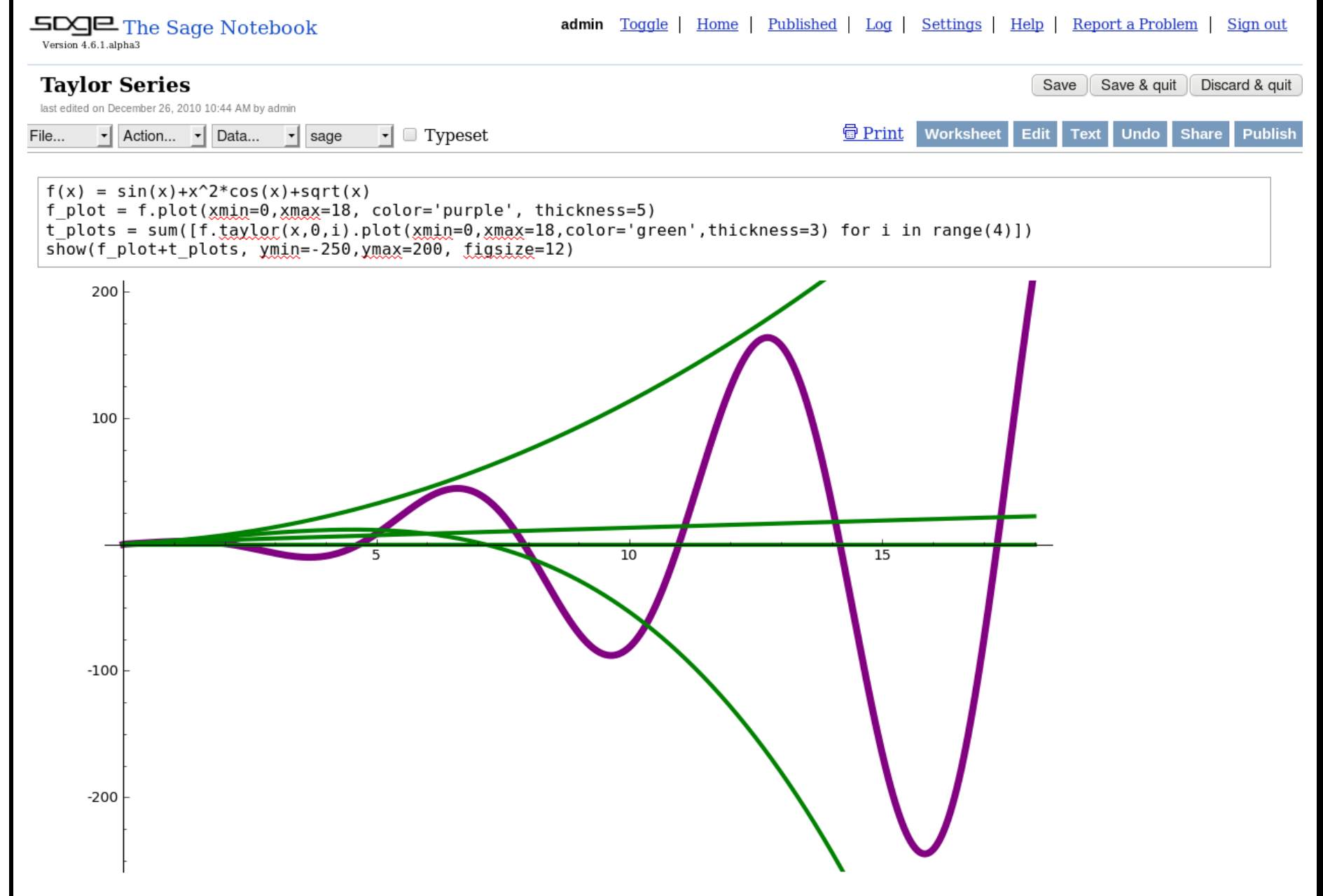
**Textbooks: Open Source Mathematics Textbooks**  
Beezer, *A First Course in Linear Algebra*  
Judson, *Abstract Algebra: Theory and Applications*  
Stein, *Elementary Number Theory*  
More...

**Resources**  
User Guides  
Subject-Specific Tutorials  
Interactive Demonstrations

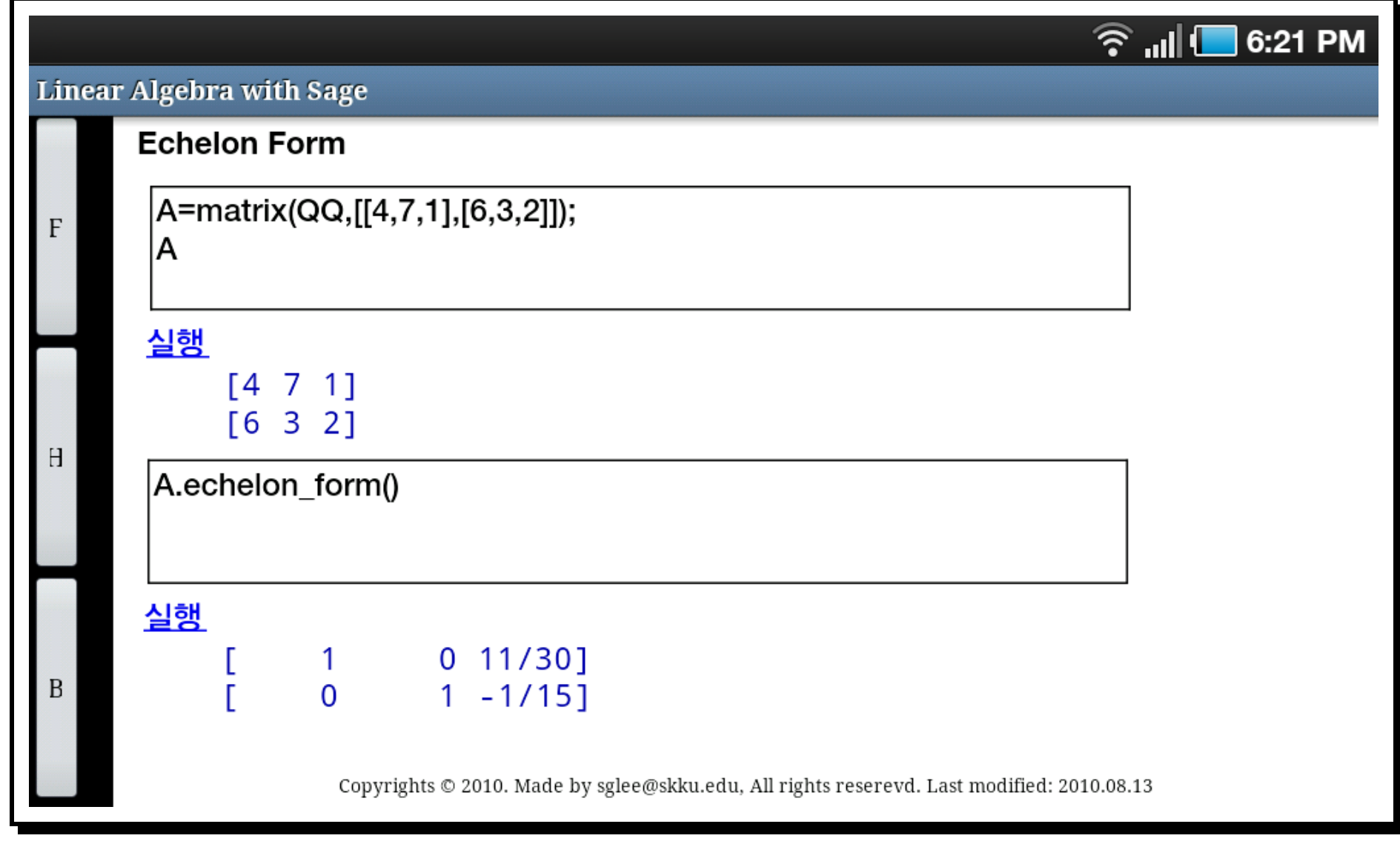
**Editorial Board**  
Review and recommend open texts in mathematics  
Organized through American Institute of Mathematics

### Open Software

Sage Notebook Interface in a Web Browser



Sage on Mobile Devices: App for Android OS  
Kyung-Won Kim, Sang-Gu Lee, SungKyunKwan U, Korea



### Open Textbooks

Linear Algebra Textbook (PDF, Print)

Section TSS.CSS Consistent Systems 68

**Sage FDV**  
**Free and Dependent Variables**  
Sage has the `.pivot()` command to quickly and easily identify the pivot columns of the reduced row-echelon form of a matrix. Notice that we do not have to row-reduce the matrix first, we just ask which columns of a matrix  $A$  would be the pivot columns of the matrix  $B$  that is row-equivalent to  $A$  and in reduced row-echelon form. By [Definition IDV \[66\]](#), the indices of the pivot columns for an augmented matrix of a system of equations are the indices of the dependent variables. And the remainder are free variables. But be careful, Sage numbers columns starting from zero and mathematicians typically number variables starting from one.

Let's reprise [Example ISS \[63\]](#).

```
sage: coeff = matrix(QQ, [
....: [1,4,0,-1,0,7,-9],
....: [2,8,-1,3,9,-13,7],
....: [0,0,2,-3,-4,12,-8],
....: [-1,-4,2,4,6,-31,37]
])
sage: const = vector([3,9,1,4])
sage: aug = coeff.augment(const)
sage: dependent = aug.pivots()
sage: dependent
[0, 2, 3]
```

So, increasing each column index by 1 gives us the set  $D$  of indices for the dependent variables. To get the free variables, we can use the following code. Study it and then read the explanation following.

```
sage: free = [index for index in range(7) if not index in dependent]
sage: free
[1, 4, 5, 6]
```

This is a Python programming construction known as a “list comprehension” but in this setting I prefer to call it “set builder notation.” Let's dissect the command in pieces. The brackets  $[ , ]$  create a new list. The items in the list will be values of the variable `index`. `range(7)` is another list, integers starting at 0 and stopping just before 7. (While perhaps a bit odd, this works very well when we consistently start counting at zero.) So `range(7)` is the list  $\{0, 1, 2, 3, 4, 5, 6\}$ . Think of these as candidate values for `index`, which are generated by `for index in range(7)`. Then we test each candidate, and keep it in the new list if it is *not* in the list `dependent`.

This is entirely analogous to the following mathematics:

$$F = \{j \mid 1 \leq j \leq 7, j \notin D\}$$

where  $F$  is free,  $j$  is index, and  $D$  is dependent, and we make the 0/1 counting adjustments. This ability to construct sets in Sage with notation so closely mirroring the mathematics is a powerful feature worth mastering.

Linear Algebra Textbook (Sage Worksheet)

### Project Activities

**CCLI Phase 2 Grant**  
September 2010 to August 2013

**Sage Notebook: Software Improvements**  
Server scalability  
Worksheet management  
Interface improvements

**Sage Library Code**  
Usability improvements

**Textbooks: Conversion to Worksheets**  
Software: conversion from  $\LaTeX$  to worksheets  
automated system using tex4ht  
Content: Live Sage code to illustrate mathematics

**Workshops: Sage Edu Days**  
Summer 2011  
Summer 2012

**Editorial Board**  
Organize and convene group  
Review and recommend open textbooks

**Test Sites**  
8 undergraduate mathematics departments  
First 4 for AY 2011-12  
All 8 for AY 2012-13

**Assessment**  
Ethnography & Evaluation Research, Univ of Colorado