

# UTMOST: Undergraduate Teaching and Learning in Mathematics with Open Software and Textbooks

Joint Mathematics Meetings, Baltimore

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# The UTMOST Project

- Development: PreTeXt is an authoring and publishing system for openly licensed textbooks
- Research: studies of student and instructor textbook use concentrating on new online formats
- UTMOST 1: Sage Cell Server, CoCalc, PreTeXt
- UTMOST 2: Pilot research study, PreTeXt development
- UTMOST 3: Large-scale research study, PreTeXt development

*PreTeXt lowers barriers to effective learning*

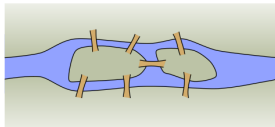
# Barrier: Cost and Portability, Access

- Print versions are very affordable ( $\sim$  \$10 – \$20)
- Open licenses  $\Rightarrow$  online versions are FREE
- No DRM (Digital Restrictions Management)
- Available: very readable on small screens (phones)
- Every student has the book
- Day Zero: every student has the book! — K. Morrison

## GRAPH THEORY

### *Investigate!*

In the time of Euler, in the town of Königsberg in Prussia, there was a river containing two islands. The islands were connected to the banks of the river by seven bridges (as seen below). The bridges were very beautiful, and on their days off, townspeople would spend time walking over the bridges. As time passed, a question arose: was it possible to plan a walk so that you cross each bridge once and only once? Euler was able to answer this question. Are you?



Attempt the above activity before proceeding

Graph Theory is a relatively new area of mathematics, first studied by the super famous mathematician Leonhard Euler in 1735. Since then it has blossomed in to a powerful tool used in nearly every branch of science and is currently an active area of mathematics research.

The problem above, known as the *Seven Bridges of Königsberg*, is the problem that originally inspired graph theory. Consider a “different” problem: Below is a drawing of four dots connected by some lines. Is it possible to trace over each line once and only once (without lifting up your pencil, starting

*Discrete Mathematics: An Open Introduction* (Levin)

# Barrier: Computing Environments

- Mathematica, Maple, Matlab: expensive, licensed
- Sage: open source, powerful, and available
- Sage Cell server
  - Zero setup, no login
  - Up 24 x 7 worldwide
  - Execute inside textbooks
  - Authors create preloaded demonstration code

The screenshot shows a web browser window with the URL `abstract.ups.edu/sage/permute-sage.html`. The page is titled "Permutation Groups and Elements" and contains the following text:

The easiest way to work with permutation group elements in Sage is to write them in cycle notation. Since these are products of disjoint cycles (which commute), we do not need to concern ourselves with the actual order of the cycles. If we write  $(1,3)(2,4)$ , we probably understand it to be a permutation (the topic of this chapter!) and we know that it could be an element of  $S_n$  or perhaps a symmetric group on more symbols than just 4. Sage cannot get started that easily and needs a bit of context, so we coerce a string of characters written with cycle notation into a symmetric group to make group elements. Here are some examples and some simple computations. Remember that Sage and your text differ on how to interpret the order of composing two permutations in a product.

The interface includes several code input boxes with "Evaluate (Sage)" buttons:

```
1 G = SymmetricGroup(5)
2 sigma = G("(1,3)(2,5,4)")
3 sigma*sigma
```

Output:  $(2,4,5)$

```
1 rho = G("(2,4)(1,3)")
2 rho^3
```

Output:  $(1,2,4,3)$

Text: "If the next three examples seem confusing, or "backwards", then now would be an excellent time to review the Sage discussion about the order of permutation composition in the subsection [Groups of symmetries](#)."

```
1 sigma*rho
```

Output:  $(1,2,4,3)$

```
1 rho*sigma
```

Output:  $(1,2,4,3)$

```
1 rho^-1*sigma*rho
```

Output:  $(1,2,4,3)$

The left sidebar contains a table of contents with sections: 5 Permutation Groups, 6 Cosets and Lagrange's Theorems, 7 Introduction to Cryptography, and 8 Algebraic Coding Theory. The bottom of the page features the MathJax logo.

*Abstract Algebra (Judson)*

# Barrier: Accessibility

- HTML version leverages open standards
- Mathematics via MathJax is screen-reader friendly
- Additional structure included to aid screen-readers
- Automatic for *every* online PreTeXt book
- “Best Practice” advice in PreTeXt *Author’s Guide*

https://sites.wcsu.edu/mbro...  
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Analyzing this we get that the most common characters are Y, D, I, O and U; the most common bigrams are DZ, ZY, YG, and OB; the most common trigrams are DZY, OBO, LDZ, and DZO. Therefore it is reasonable to assume that DZY is the, Y is e, and D is t. So when this was enciphered we have to of had

$$\begin{aligned} 24 &= m \cdot 4 + s \pmod{26} \\ 3 &= m \cdot 19 + s \pmod{26} \end{aligned}$$

Subtracting the second equation from the first gives

$$21 = m \cdot -15 \pmod{26}$$

or

$$21 = m \cdot 11 \pmod{26}.$$

Looking at the multiplication table modulo 26 we can see that  $m = 9$  since  $9 \cdot 11 = 21 \pmod{26}$ . Substituting  $m = 9$  into the first equation above we get

$$24 = 9 \cdot 4 + s \pmod{26}$$

which simplifies to

$$24 - 10 = s \pmod{26}$$

so that  $s = 14$ . We can then get the inverse keys  $m^{-1} = 3 \pmod{26}$  and  $-m^{-1}s = 10 \pmod{26}$ . Using these with the affine cipher cell we get the deciphered message:

thisi sthef irsta ffine ciphe rness ageth atwev illde crypt  
itwas encip hered witha keyof orthe multi plier andfo  
urtee nfort heshi ftsin ceiti samon salph abeti cweca natia  
okitw itiba sicfr eqaen cyana lysis butee canal sousee thata  
nalys istor ecove rtheo rigin alkey sotha tweca ndeci phert  
heses sage

Or, in a more readable form

“this is the first affine cipher message that we will decrypt...”

[Checkpoint 6.1.21](#)

*Cryptology Through History  
and Inquiry (Rocca)*

# Effective Learning: Embedded WeBWork Problems

- WeBWork: system for interactive exercises
- Open source, NSF support
- Author problems in PreTeXt
- Embed problems in output
- Static versions for print
- Extensive practice
- Immediate feedback
- Patient feedback

The screenshot shows a web browser window with the URL `sp19.pcc.edu/math/orcca/`. The page has a navigation menu on the left with sections 5 through 7. Section 5, "Systems of Linear Equations", is highlighted. The main content area displays "Example 5.3.3 Scaling One Equation" with a system of equations: 
$$\begin{cases} 3x - 4y = 2 \\ 5x + 8y = 18 \end{cases}$$
 Below the equations is an "Explanation" section and a "Checkpoint 5.3.4" which asks to solve a similar system: 
$$\begin{cases} 5x + 4y = -7 \\ 5x + 2y = -1 \end{cases}$$
 A text input field is provided for the answer. Below the input field is a "Solution:" section with a light blue background containing a step-by-step explanation: 

1. We subtract the two equations, which will cancel the terms in involving  $x$  and give  $4y - 2y = -7 - (-1)$ .
2. This gives  $y = -3$ .
3. Now that we have  $y$ , we find  $x$  using either equation. Let's use the first:  $5x - 12 = -7$ , so  $x = 1$ .
4. The solution to the system is  $(1, -3)$ . It is left as an exercise to check. Please also note that you may have solved this problem a different way.

 At the bottom of the solution box are two buttons: "Check answer" and "Show correct answer".

*ORCCA: Open Resources for  
Community College Algebra  
(Portland CC Faculty)*

# Effective Learning: Reading Questions

- Promotes active reading
- Promotes daily preparation
- Helps instructor plan use of classroom time
- Students answer questions directly in the book
- Instructor's responses are returned in student's book

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## 4.3.4 Reading Questions

1. For a continuous function  $f$  that is defined on the interval  $[a, b]$ , what does the definite integral  $\int_a^b f(x) dx$  measure?  
*My answer →*
2. In your own words, what is the difference between a definite integral and a Riemann sum?  
*My answer →*
3. What are two rules for definite integrals that are similar to rules for derivatives? What do these rules tell us?

**Addition:**

$$\int_a^b f(x) + g(x) dx = \int_a^b f(x) dx + \int_a^b g(x) dx$$

**Multiplication by a constant:**

[\[delete\]](#) [\[edit\]](#) [\[how to write math\]](#)

### 4.3.5 Summary

- Any Riemann sum of a continuous function  $f$  on an interval  $[a, b]$  provides an estimate of the net signed area bounded by the function and the horizontal axis on the interval. Increasing the number of subintervals in the Riemann sum improves the accuracy of this estimate, and letting the number of subintervals increase without bound results in the values of the corresponding Riemann sums approaching the exact value of the enclosed net signed area.
- When we take the limit of Riemann sums, we arrive at what we call the definite integral of  $f$  over the interval  $[a, b]$ . In particular, the symbol  $\int_a^b f(x) dx$  denotes the definite integral of  $f$  over  $[a, b]$ , and this quantity is defined by the equation

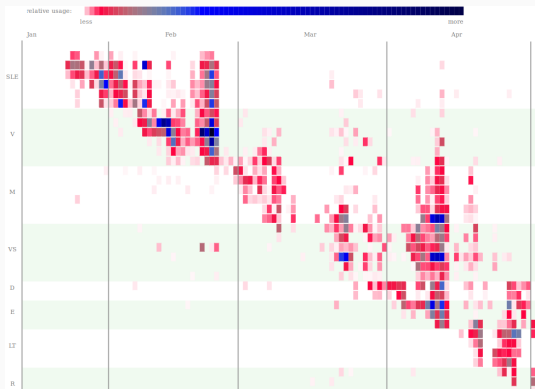
$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i^*) \Delta x.$$

Feedback  
Authored in PreTeXt  
POWERED BY MathJax

## Active Calculus (Boelkins)

# Effective Learning: Better Research

- PreTeXt enforces explicit structure, mirrored in the HTML version
- With logins and Javascript, analyze reader activity
- Summarize reading and homework activity for instructors?



Heat Map: Across - Days; Down - Sections  
*A First Course in Linear Algebra* (Beezer)



The Research Component

# TEXTBOOK USE



SCHOOL OF  
EDUCATION  
UNIVERSITY OF MICHIGAN

# RESEARCH QUESTIONS

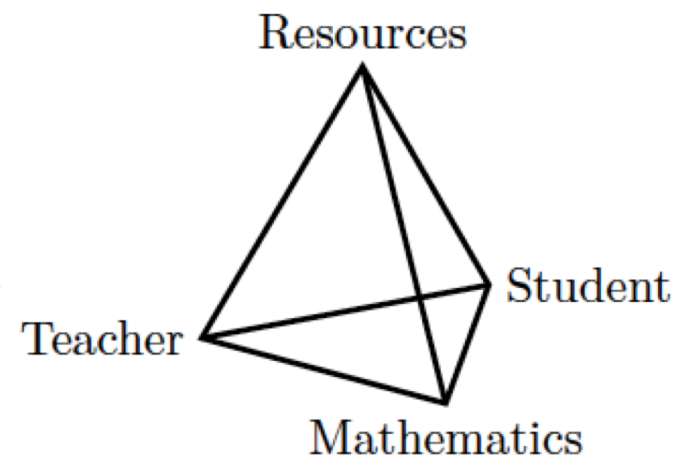
1. How do instructors and students use these textbooks as they teach and learn?
2. Are there differences in use between dynamic and PDF formats?
3. How can the features in the textbooks be used to alter instruction?

# PHASES

- Pilot study (F16-W18): instrument development and testing with 11 instructors, seven case studies, two textbooks
  - Beezer's [First Course in Linear Algebra](#)
  - Judson's [Abstract Algebra: Theory and Applications](#)
- Current study: Goal 49 sections, 40 instructors, nine case studies, and a third textbook:
  - Boelkin's [Active Calculus](#)

# THEORETICAL FRAMEWORK

- Documentational approach
  - An artifact (e.g., a textbook) becomes an instrument once a scheme of use has been defined
  - Resources (a set of artifacts gathered for a particular purpose) become documents once a scheme of use has been defined
  - Schemes of use include rules of action and information about when/why rules are enacted
- Instructional tetrahedron
  - Interactions are mediated by textbook and resources



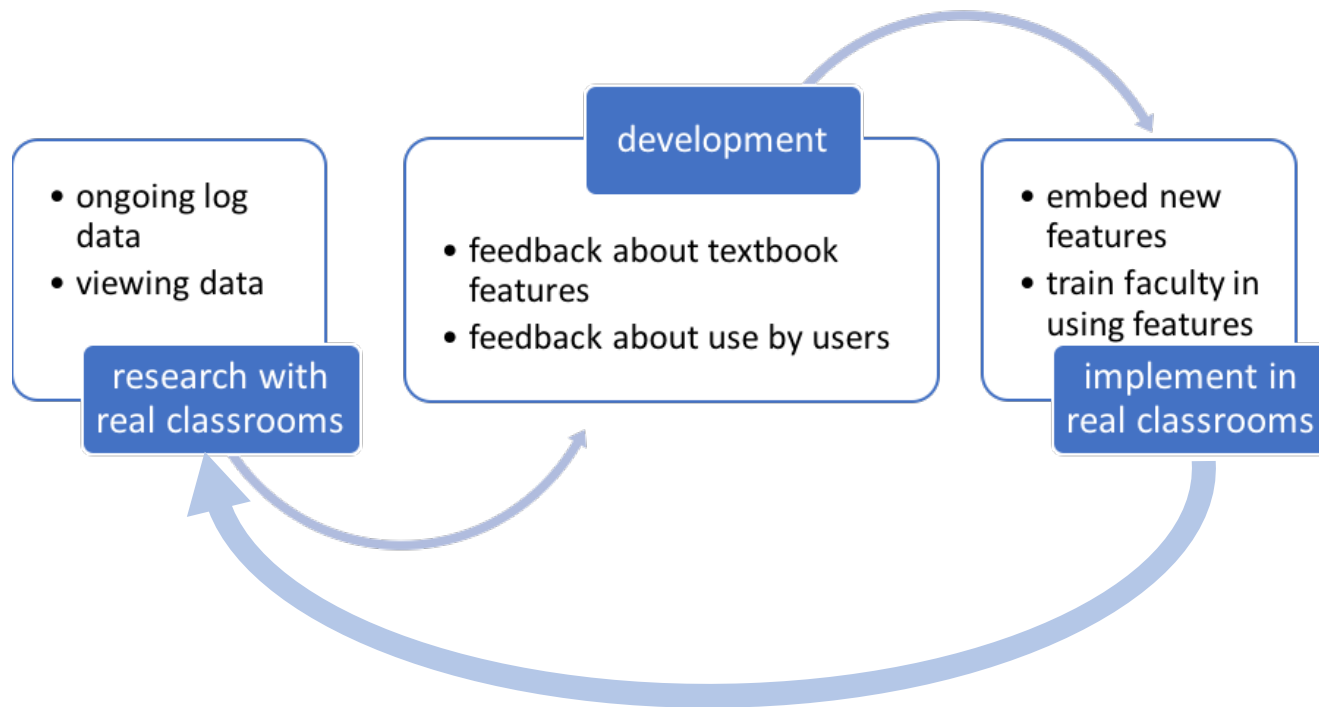
(Guedet & Trouche, 2009; Pepin et al, 2015; Rezat, 2006)

# MIXED METHODS EMBEDDED DESIGN

- Qualitative and quantitative data collected simultaneously
  - Ongoing data collection of textbook use via bi-weekly surveys (Logs), viewing data for students and instructor, artifacts
  - Surveys: attitudes and knowledge
  - Student test of knowledge
  - Repeated site visits (interviews, observations, focus groups) before and after Collaborative Workshop
- Recruited so far:
  - 33 instructors in 18 states
  - ~700 students

# VARIOUS CYCLES OF RESEARCH, DEVELOPMENT, AND IMPLEMENTATION

- Data collection: Seven semesters, each with five to nine sections:



# DATA COLLECTION PLAN

	Beginning of Term	Week in the term						End of Term
		2	4	6	8	10	12	14
Teacher surveys	X							
Teacher logs		X	X	X	X	X	X	
Site visits for nine instructors: Three teacher interviews								
I1: Planning								
I2: Enacting					← X →			
I3: Reflecting								
Class observations								
Student focus groups								
Computer-generated data of teacher and student textbook use		~~~~~						
Student logs		X	X	X	X	X	X	
Student survey					X			
Student tests		X						X
Student grades								X

Collaborative workshop with instructors, authors, developers, researchers, in intervening summer for a subset of six instructors



# ANALYSIS

- Ongoing analysis within a semester:
  - Student and instructor logs using language processing to inform log design and needs for feature modification
- Analysis across semesters:
  - Resource use, schemes of use, changes due to intervening Collaborative Workshop
  - Reports for individual faculty
  - Aggregate reports for research team
- Aggregate across all data collected:
  - Surveys, tests, and grades → what is the impact for students?
  - Resource use, classroom data → what is the impact on practice?



# UNDERGRADUATE TEACHING AND LEARNING IN MATHEMATICS WITH OPEN SOFTWARE AND TEXTBOOKS

THANK YOU!

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[utmost.aimath.org](http://utmost.aimath.org)

[pretextbook.org](http://pretextbook.org)

American Institute of Mathematics (DUE–1626455, 1821706): Rob Beezer, David Farmer, Kent Morrison

Stephen F. Austin State University (DUE–1625223, 1821329): Thomas Judson

University of Colorado Boulder (DUE–1624998, 1821114): Susan Lynds

University of Michigan (DUE–1624634, 1821509): Vilma Mesa, Angeliki Mali, Yannis Liakos



# REFERENCES

- Gueudet, G., & Trouche, L. (2009). Towards new documentation systems for mathematics teachers? *Educational Studies in Mathematics*, 7, 199–218.
- Pepin, B., Guedet, G., Yerushalmy, M., Trouche, L., & Chazan, D. (2015). E-textbooks in/for teaching and learning mathematics: A potentially transformative educational technology. In L. English & D. Kirshner (Eds.), *Third handbook of international research in mathematics education* (pp. 636–661). London: Taylor Francis.
- Rezat, S. (2006). The structure of German mathematics textbooks. *Zentralblatt für Didaktik der Mathematik*, 32, 482-488.