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MAA Session on Research in Undergraduate Mathematics Education

INSTRUCTORS AND STUDENTS' USES OF DYNAMIC TEXTBOOKS: WHAT IS NEW?

SCHOOL OF EDUCATION UNIVERSITY OF MICHIGAN

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OVERVIEW

Free, open source, dynamic textbooks for university mathematics courses are becoming available in undergraduate mathematics classrooms.

Exploratory study to investigate instructor, and student, uses of two free, open source textbooks, Rob Beezer's *First Course in Linear Algebra* , and Tom Judson's *Abstract Algebra: Theory and Applications*.

Research questions

1. What are the features of the textbooks?
2. Do instructors and students take advantage of the textbook features? If so, how?

EDUCATION

OUTLINE

1. What are the features of the textbooks?
   - Textbook analysis of First Course in Linear Algebra
2. Do students take advantage of the textbook features? If so, how?
   - Student usage from bi-weekly logs and automatically collected data
3. Do instructors take advantage of the textbook features? If so, how?
   - Instructor usage from observations and interviews

EDUCATION

1. WHAT ARE THE FEATURES OF THE TEXTBOOKS?

- Dynamic features: built-in digital features offering new types of user interface in terms of navigation, computation, and text modification
  - Table of contents, index, prev/up/next buttons, search engine, knowls & cross referencing, Sage cells, open source.
- Scope of contents (Usiskin, 2017): textbook content and author intention
  - Definition, end of chapter question/exercise, example, hint or worked out answer, introductory summary of section, metaphor, proof, purpose, theorem.
- Mathematical practices (Usiskin, 2017): ways of doing math
  - Deduction—the standard by which we decide whether a statement is true or not
  - Representation—the result of the move from one mode of describing a piece of mathematics to another mode
  - Symbolization—vocabulary and notation

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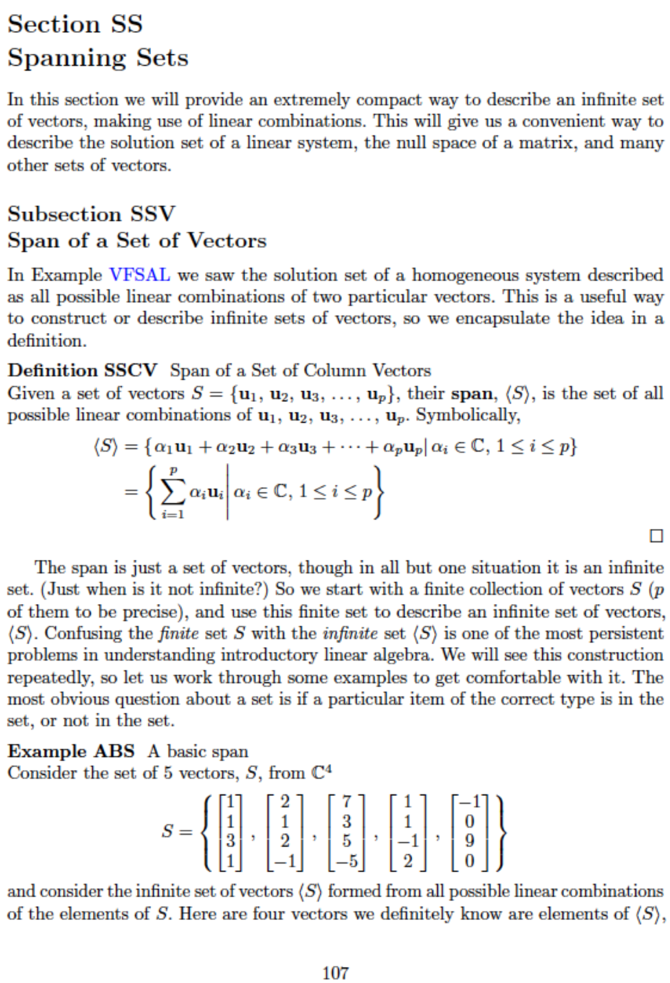
1. WHAT ARE THE FEATURES OF THE TEXTBOOKS?



Section SS
Spanning Sets

In this section we will provide an extremely compact way to describe an infinite set of vectors, making use of linear combinations. This will give us a convenient way to describe the solution set of a linear system, the null space of a matrix, and many other sets of vectors.

Subsection SSV
Span of a Set of Vectors

In Example VFSAL we saw the solution set of a homogeneous system described as all possible linear combinations of two particular vectors. This is a useful way to construct or describe infinite sets of vectors, so we encapsulate the idea in a definition.

Definition SSCV Span of a Set of Column Vectors
Given a set of vectors $S = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \ldots, \mathbf{u}_p\}$, their **span**, $\langle S\rangle$, is the set of all possible linear combinations of $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \ldots, \mathbf{u}_p$. Symbolically,

$$\langle S\rangle = \{\alpha_1\mathbf{u}_1 + \alpha_2\mathbf{u}_2 + \alpha_3\mathbf{u}_3 + \cdots + \alpha_p\mathbf{u}_p \mid \alpha_i \in \mathbb{C},\ 1 \le i \le p\}$$
$$= \left\{\sum_{i=1}^{p} \alpha_i\mathbf{u}_i \,\middle|\, \alpha_i \in \mathbb{C},\ 1 \le i \le p\right\}$$

□

The span is just a set of vectors, though in all but one situation it is an infinite set. (Just when is it not infinite?) So we start with a finite collection of vectors S (p of them to be precise), and use this finite set to describe an infinite set of vectors, $\langle S\rangle$. Confusing the *finite* set S with the *infinite* set $\langle S\rangle$ is one of the most persistent problems in understanding introductory linear algebra. We will see this construction repeatedly, so let us work through some examples to get comfortable with it. The most obvious question about a set is if a particular item of the correct type is in the set, or not in the set.

Example ABS A basic span
Consider the set of 5 vectors, S, from $\mathbb{C}^4$

$$S = \left\{\begin{bmatrix}1\\1\\3\\1\end{bmatrix}, \begin{bmatrix}2\\1\\2\\-1\end{bmatrix}, \begin{bmatrix}7\\3\\5\\-5\end{bmatrix}, \begin{bmatrix}1\\1\\-1\\2\end{bmatrix}, \begin{bmatrix}-1\\0\\9\\0\end{bmatrix}\right\}$$

and consider the infinite set of vectors $\langle S\rangle$ formed from all possible linear combinations of the elements of S. Here are four vectors we definitely know are elements of $\langle S\rangle$,

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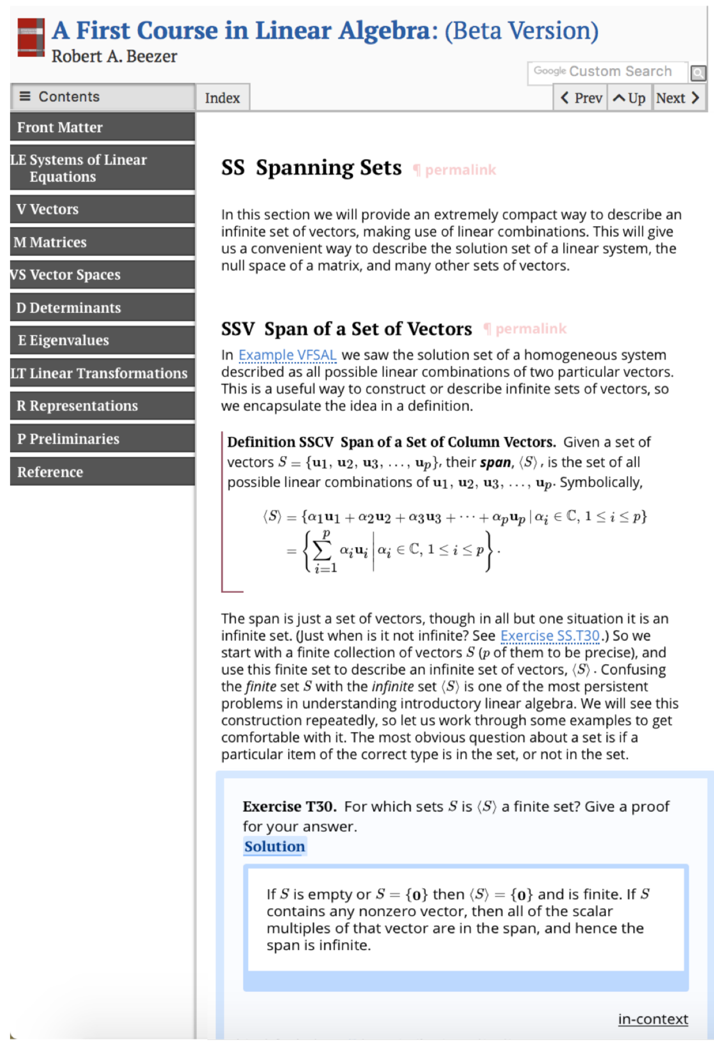
Figure 1: PDF version (Beezer, 2015)



A First Course in Linear Algebra: (Beta Version)
Robert A. Beezer

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Front Matter
LE Systems of Linear Equations
V Vectors
M Matrices
VS Vector Spaces
D Determinants
E Eigenvalues
LT Linear Transformations
R Representations
P Preliminaries
Reference

SS Spanning Sets ¶ permalink

In this section we will provide an extremely compact way to describe an infinite set of vectors, making use of linear combinations. This will give us a convenient way to describe the solution set of a linear system, the null space of a matrix, and many other sets of vectors.

SSV Span of a Set of Vectors ¶ permalink

In Example VFSAL we saw the solution set of a homogeneous system described as all possible linear combinations of two particular vectors. This is a useful way to construct or describe infinite sets of vectors, so we encapsulate the idea in a definition.

Definition SSCV Span of a Set of Column Vectors. Given a set of vectors $S = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \ldots, \mathbf{u}_p\}$, their ***span***, $\langle S\rangle$, is the set of all possible linear combinations of $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \ldots, \mathbf{u}_p$. Symbolically,

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$$= \left\{\sum_{i=1}^{p} \alpha_i\mathbf{u}_i \,\middle|\, \alpha_i \in \mathbb{C},\ 1 \le i \le p\right\}.$$

The span is just a set of vectors, though in all but one situation it is an infinite set. (Just when is it not infinite? See Exercise SS.T30.) So we start with a finite collection of vectors S (p of them to be precise), and use this finite set to describe an infinite set of vectors, $\langle S\rangle$. Confusing the *finite* set S with the *infinite* set $\langle S\rangle$ is one of the most persistent problems in understanding introductory linear algebra. We will see this construction repeatedly, so let us work through some examples to get comfortable with it. The most obvious question about a set is if a particular item of the correct type is in the set, or not in the set.

Exercise T30. For which sets S is $\langle S\rangle$ a finite set? Give a proof for your answer.
Solution

If S is empty or $S = \{\mathbf{0}\}$ then $\langle S\rangle = \{\mathbf{0}\}$ and is finite. If S contains any nonzero vector, then all of the scalar multiples of that vector are in the span, and hence the span is infinite.

in-context

Figure 2: HTML version (Beezer, 2017, adapted from O'Halloran et al., 2018)

EDUCATION

1. WHAT ARE THE FEATURES OF THE TEXTBOOKS?

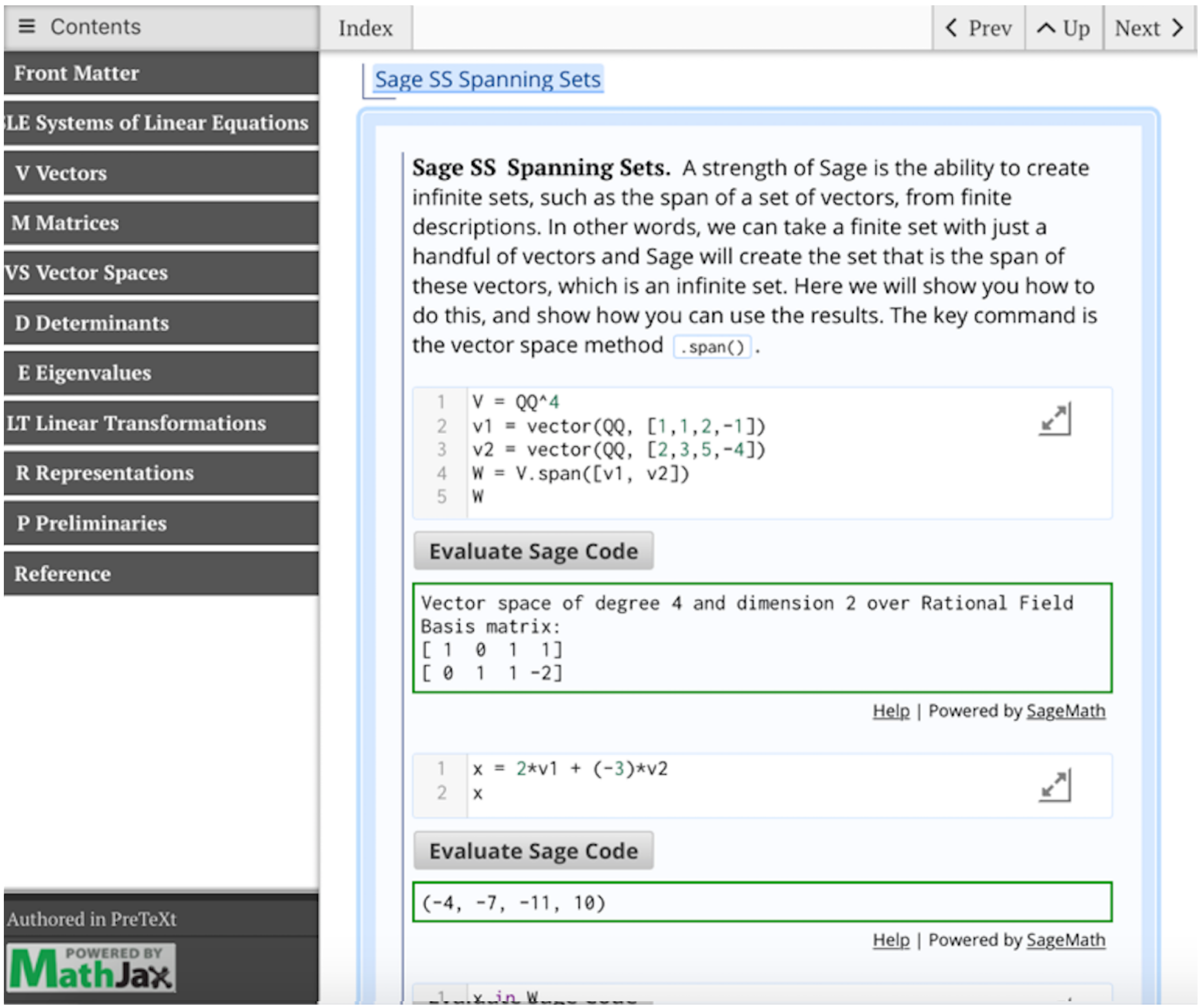

Figure 3: Sage cells in HTML version (Beezer, 2017)

EDUCATION

DESIGN

- Seven teachers and their students:

Version	Textbook	Course	Instructor	# of Students
HTML	Beezer	Linear Algebra	T1	29
			T4	12
			T6	22
	Judson	Abstract Algebra	T3	12
			T5	27
PDF	Beezer	Linear Algebra	T7	19
Bounded	Strang	Abstract Algebra	T2	37

- Six institutions, four states:

  Large research, mid-size regional, and small state, universities in California, Michigan, New York and Texas.

- Ongoing data collection (data analytics and logs) and
  one week site visits (interviews, observations, focus groups, documents)

EDUCATION

2. DO STUDENTS TAKE ADVANTAGE OF THE TEXTBOOK FEATURES? IF SO, HOW?

A user's actions, such as revealing a solution by clicking on a knowl, can be recorded along with the time spent on that part of the textbook.

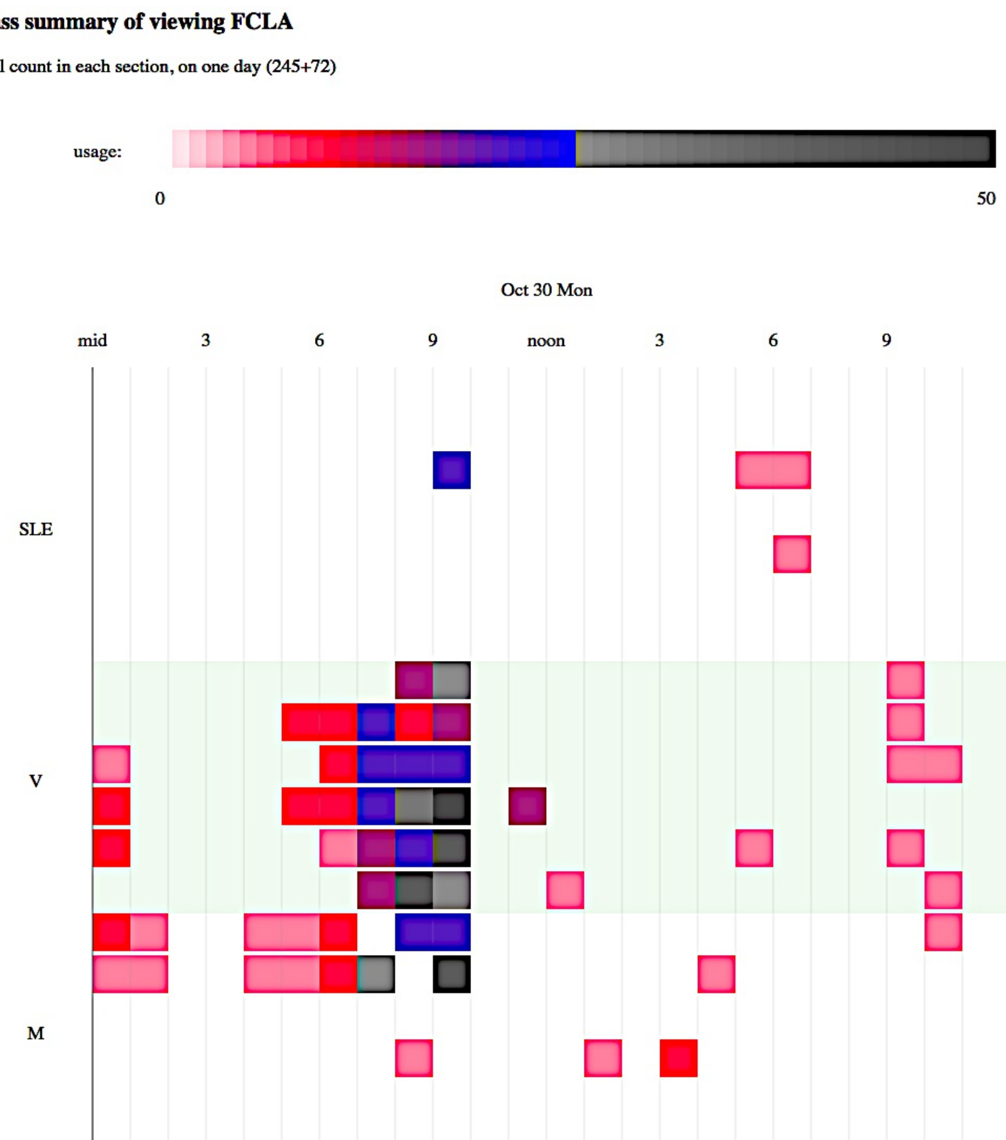

Figure 5: Heat map of textbook use.

2. DO STUDENTS TAKE ADVANTAGE OF THE TEXTBOOK FEATURES? IF SO, HOW?

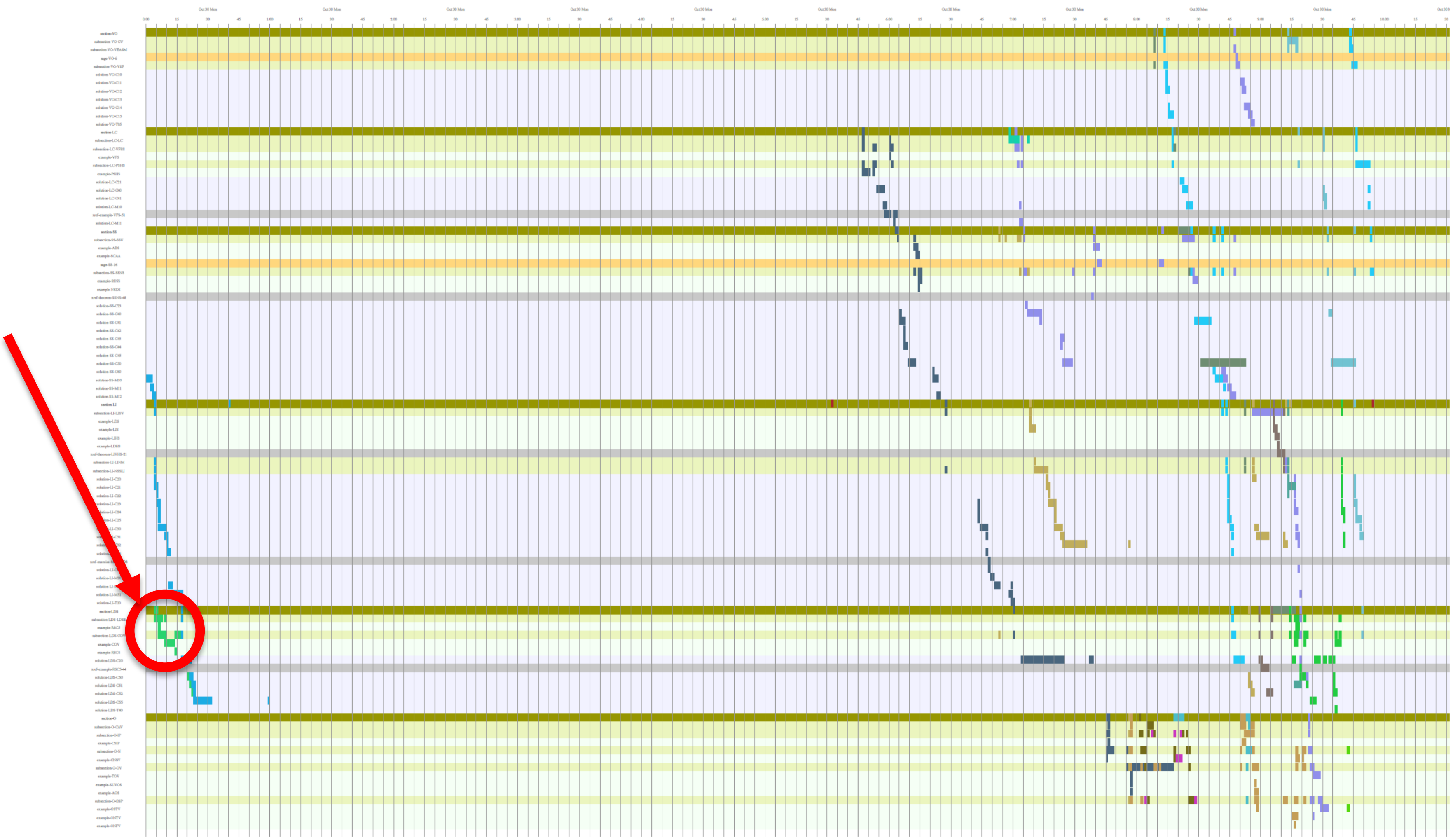

Figure 6: Data organized by individual user, with resolution to the minute and at the level of examples, figures, theorems, exercises, solutions, and other components of the textbook.

EDUCATION

2. DO STUDENTS TAKE ADVANTAGE OF THE TEXTBOOK FEATURES? IF SO, HOW?

Log question

Please identify the color that represents you, and tell us what you were doing with a couple of sections represented by a couple of rectangles. Start your response by stating your color, the section, and the time shown in the rectangle.

"Green, section LDS, 0:00 -1:00. I studied for my quiz by studying the examples in the book."

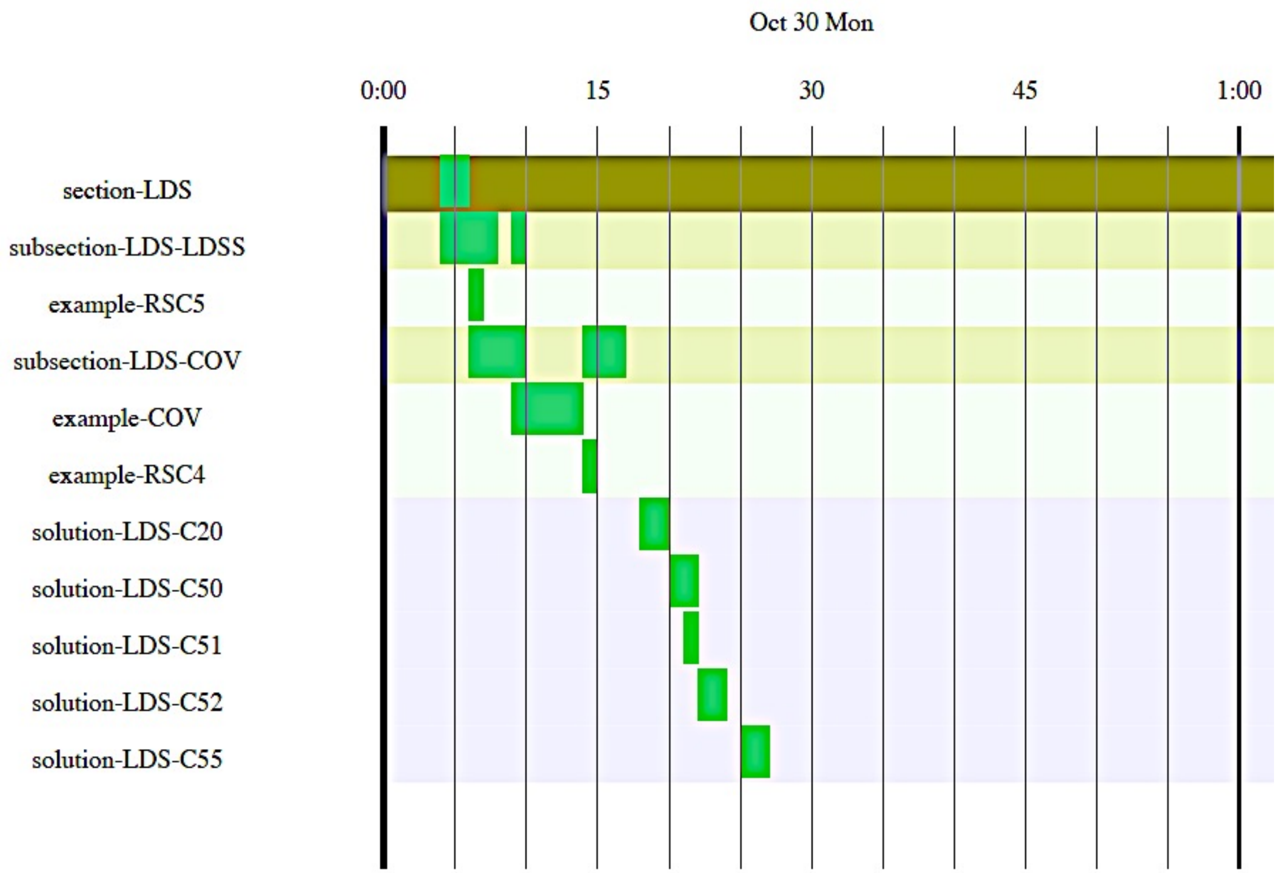

Figure 7: Data of a user, with resolution to the minute and at the level of theorems, examples, solutions of the textbook.

2. DO INSTRUCTORS TAKE ADVANTAGE OF THE TEXTBOOK FEATURES? IF SO, HOW?

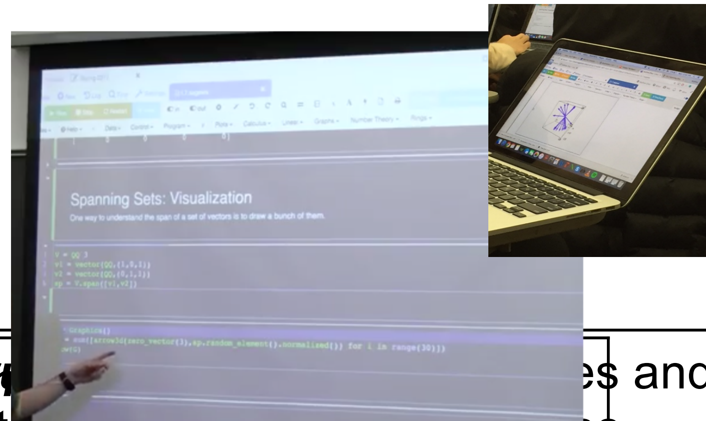

Figure 9: Lecture notes embedded in Sage worksheet and simultaneous student work.

Instr… es and constraints of the set of resources influences the teacher/user

An instructor ⟷ A set of resources

"The geometric interpretation in $\mathbb{R}^3$ with more than two vectors linearly dependent better reveals the concept of linear dependence [than the technical definition of linear combinations being zero.]"

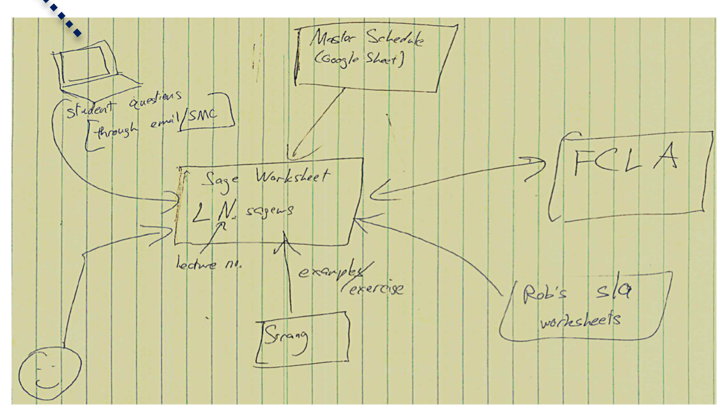

Figure 8: Instructor set of resources.

 The schematic representation of the processes of *instrumentation* and *instrumentalization* is adapted from Gueudet and Trouche (2009).

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STUDENT USES OF DYNAMIC TEXTBOOKS: WHAT IS NEW?

- There is some resistance to moving to the use of a dynamic textbook by a couple of students, evident from their use of bounded/PDF versions.
- Students report they value the navigation features, and the accessibility of the textbook.
- Student textbook uses, as reported in logs, are congruous with uses of bounded textbooks; students view problems, examples, definitions, and theorems in order to understand the material, to study for midterms, and to do homework.

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INSTRUCTOR USES OF DYNAMIC TEXTBOOKS: WHAT IS NEW?

- Instructors use textbooks accordingly to:
  - their perceptions of teaching and learning at university level
  - knowledge of availability of, and familiarity with, dynamic features
- Instructors take advantage of the textbook features only when those can be seamlessly integrated into their usual practices.
  - They create their lecture notes attending to the sequencing of topics presented in the textbook and maintaining the notation, definitions, and theorems.
  - They adjust their use of technology in the classroom to ways they have been using it in past.

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UNDERGRADUATE TEXTBOOKS IN MATHEMATICS WITH OPEN SOFTWARE AND TEXTBOOKS

THANK YOU!

Collaborators:

Tom Judson	Stephen F Austin State U
Rob Beezer	University of Puget Sound
David Farmer	American Institute of Mathematics
Susan Lynds	University of Colorado at Boulder
Kent Morrison	American Institute of Mathematics

utmost.aimath.org

mathbook.pugetsound.edu

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